

Bethany Lutheran School  
Long Beach, California

Mathematics  
Content Standards

**Introduction**

*A study of mathematics enables us to appreciate the orderliness and wisdom of God's creation. To each of His creatures He has given certain attributes and abilities. To the one created in "His own image" He has given the ability to reason. The extensive nature of mathematical development—a purely human endeavor—is evidence of the depth of this ability and the greatness of God's creation.*

*To each of His children God gives different gifts. As Lutheran educators we recognize the difference in these gifts along with the constancy of His love. Through the Gospel of Jesus Christ, we are able not only to provide for individual differences but also to cherish each individual as a child of God as we guide, discipline, forgive, and instruct. Because it is so easy for some students and so very difficult for others, the study of mathematics gives us great opportunity for the demonstration of our Christian love and caring.*

*Everyone needs some knowledge of mathematics to function well in today's society. As the indispensable language of technology, business, and finance, mathematics provides essential problem-solving skills that can be applied to a wide range of scientific disciplines and everyday situations. Every citizen needs mathematical competence to succeed and contribute to society. Perhaps more true now than at any time in the past, a knowledge and appreciation of mathematics is critical to intellectual development.*

# KINDERGARTEN

By the end of kindergarten, students understand the consistency of small numbers, quantities and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense about properties and patterns.

## NUMBER SENSE

1. Students understand the relationship between numbers and quantities, i.e., that a set of objects has the same number of objects in different situations, regardless of its position or arrangement.
  - 1.1. compare two or more sets (up to 10 objects in each group), and identify which set is equal to, more than, or less than the other
  - 1.2. count, recognize, represent, name and order numbers (*to 31*) using objects
  - 1.3. know that the larger numbers describe sets with more objects in them than smaller numbers
  - 1.4. *count by 1's, 5's, and 10's to 100*
2. Students understand and describe simple addition and subtraction situations.
  - 2.1 use concrete objects to determine the answers to addition and subtraction problems (for two numbers each less than 10)
  - 2.2 *count backwards from 20 to 1*
3. Students use estimation strategies in computation and problem solving that involve numbers that use the ones and tens places.
  - 3.1 recognize when an estimate is reasonable

## ALGEBRA AND FUNCTIONS

1. Students sort and classify objects.
  - 1.1. identify, sort and classify objects by attribute and identify objects that do not belong to a particular grouping (e.g., all these balls are green, those are red)
2. *Students identify the missing addend and subtrahend up to 10*
3. *Understand the meaning of the symbols*

## MEASUREMENT AND GEOMETRY

1. Students understand that there are properties such as length, weight, capacity and time and that comparisons can be made by using these properties.
  - 1.1. compare the length, weight and capacity of objects by making direct comparisons or using reference objects (e.g., shorter/longer/taller, lighter/heavier, which holds more?) *including tools that measure weight and length (e.g. scale, rulers)*
  - 1.2. demonstrate understanding of concepts of time (e.g., morning, afternoon, evening, day, yesterday, tomorrow, week, year) including tools that measure time (e.g., clock, calendar)
  - 1.3. name the days of the week *and the months of the year*
  - 1.4. identify the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock, bed time is 8 o'clock at night)
2. Students identify common geometric objects in their environment and describe their features.
  - 2.1. identify and describe common geometric objects (e.g., circle, triangle, square, rectangle, cube, sphere, cone)
  - 2.2. compare familiar plane and solid objects by common attributes (e.g., position, shape, size, roundness, number of corners)

## **STATISTICS, DATA ANALYSIS and PROBABILITY**

1. Students collect information about objects and events in their environment.
  - 1.1. pose information questions, collect data and record the results using objects, pictures and picture graphs
  - 1.2. identify, describe and extend simple patterns involving shape, size, or color such as circle, triangle, or red, blue

## **MATHEMATICAL REASONING**

1. Students make decisions about how to set up a problem.
  - 1.1. decide about the approach, materials and strategies to use
  - 1.2. use tools and strategies such as manipulatives or sketches to model problems
2. Students solve problems in reasonable ways and justify reasoning.
  - 2.1. explain the reasoning used with concrete objects and pictorial representations
  - 2.2. make precise calculations and check the validity of the results from the context of the problem

# GRADE 1

By the end of first grade, students understand and use the concept of "ones" and "tens" in the place value number system. They add and subtract small numbers with ease. They measure with simple units and locate objects in space. They describe data and analyze and solve simple problem situations.

## NUMBER SENSE

1. Students understand and use numbers up to 100.
  - 1.1. count, read and write whole numbers to 100
  - 1.2. compare and order whole numbers to 100 using the symbols for less than, equal to, or greater than (<, =, >)
  - 1.3. represent equivalent forms of the same number through the use of physical models, diagrams and number expressions (to 20) (e.g., 8 can be represented as  $4 + 4$ ,  $5 + 3$ ,  $2 + 2 + 2 + 2$ ,  $10 - 2$ ,  $11 - 3$ )
  - 1.4. count and group objects into ones and tens (e.g., 3 groups of ten and 4 more is 34 or  $30 + 4$ )
  - 1.5. identify and know the value of coins and show different combinations of coins that equal the same value
  - 1.6. *identify numbers as even or odd (to 20)*
  - 1.7. *identify ordinal numbers first through sixth*
  - 1.8. *identify and illustrate fractions  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$*
2. Students demonstrate the meaning of addition and subtraction and use these operations to solve problems.
  - 2.1 know the addition facts (sums to 20) and the corresponding subtraction facts, and commit them to memory
  - 2.2 use the inverse relationship between addition and subtraction to solve problems
    - 2.1. identify one more than, one less than, ten more than, ten less than a given number
    - 2.2. count by 2s, 5s and 10s with numbers to 100
  - 2.3. show the meaning of addition (putting together, increasing) and subtraction (taking away, comparing, finding the difference)
  - 2.4. solve addition and subtraction problems with one- and two-digit numbers (e.g.,  $5 + 58 = \underline{\quad}$ )
  - 2.5. find the sum of three one-digit numbers
3. Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, and hundreds places.
4. 3.1 make reasonable estimates when comparing larger or smaller numbers

## ALGEBRA AND FUNCTIONS

1. Students use number sentences to solve problems.
  - 1.1. write and solve number sentences from problem situations that express relationships involving addition and subtraction
  - 1.2. understand the meaning of the symbols +, -, =
  - 1.3. create problem situations that could lead to given number sentences involving addition and subtraction

## MEASUREMENT AND GEOMETRY

1. Students use direct comparison and non-standard units to describe the measurements of objects.
  - 1.1. compare the length, weight and volume of two or more objects using direct comparison or a non-standard unit
  - 1.2. tell time to the nearest half hour and compare time related to events (e.g., before/after, shorter/longer)
2. Students identify common geometric figures, classify them by common attributes and describe their relative position/or their location in space.
  - 2.1. identify, describe, and compare triangles, rectangles, squares and circles, including the faces of three-dimensional objects
  - 2.2. classify *and identify* familiar plane and solid objects by common attributes like color, position, shape, size, roundness, number of corners and explain which attributes are being used for classification

- 2.3. give and follow directions about location
- 2.4. describe and arrange objects in space in terms of proximity, position and direction (e.g., near, far, below, above, up, down, behind, in front of, next to, left/right)
- 2.5. *identify congruent shapes*

## **STATISTICS, DATA ANALYSIS and PROBABILITY**

1. Students organize, represent and compare categorical data on simple graphs and charts.
  - 1.1. sort objects and data by common attributes and describe the groups formed using categorical labels
  - 1.2. represent and compare data (e.g., largest, smallest, most often, least often), using pictures, bar graphs, tally charts and picture graphs
2. Students sort objects, and create and describe patterns involving numbers, shape, size, rhythm, or color.
  - 2.1. describe, extend and explain how to get to the next element in simple repeating patterns (e.g., rhythmic, numeric, color and shape patterns)

## **MATHEMATICAL REASONING**

1. Students make decisions about how to set up a problem.
  - 1.1. decide about the approach, materials and strategies to use
  - 1.2. use tools such as manipulatives or sketches to model problems
2. Students solve problems and justify their reasoning.
  - 2.1. Explain the reasoning used and justify the procedures selected
  - 2.2. make precise calculations and check the validity of the results from the context of the problem
3. Students note connections between one problem and another.

# GRADE 2

By the end of second grade, students understand place value and number relationships as they add and subtract and they use simple concepts of multiplication. They measure quantities with appropriate units. They classify and see relationships among shapes by paying attention to the elements that compose them. They collect and analyze data and verify answers.

## NUMBER SENSE

1. Students understand the relationship among numbers, quantities and place value in whole numbers up to 1000.
  - 1.1. count, read, write whole numbers to 1,000 and identify the place value for each digit
  - 1.2. use words, models and expanded form to represent numbers (to 1,000)
  - 1.3. order and compare whole numbers up to 1,000 using the symbols  $<$ ,  $=$ ,  $>$
2. Students estimate, calculate and solve problems involving addition and subtraction of two- and three-digit numbers.
  - 2.1. understand and use the inverse relationship between addition and subtraction (e.g., an opposite number sentence for  $8 + 6 = 14$  is  $14 - 6 = 8$ ) to solve problems and check solutions
  - 2.2. find the sum or difference of two whole numbers up to three digits long ***with and without renaming***
  - 2.3. use mental arithmetic to find the sum or difference of two 2-digit numbers
  - 2.4. ***identify ordinal numbers through twentieth***
3. Students model and solve simple problems involving multiplication and division.
  - 3.1 use repeated addition, arrays, counting by multiples to do multiplication
  - 3.2 use repeated subtraction, equal sharing and forming equal groups to do division with remainders
  - 3.3 know the multiplication tables of ***0s, 1s, 2s, 5s, 10s, and 100s*** (to "times 10") and commit to memory
4. Students understand that fractions and decimals can refer to parts of a set and parts of a whole.
  - 4.1 recognize, name and compare unit fractions up to  $1/12$
  - 4.2 recognize fractions of a whole and parts of a group (e.g.,  $1/4$ th of a pie,  $2/3$ rd of 15 balls)
  - 4.3 know that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one
  - 4.4 ***identify and write mixed numbers***
5. Students model and solve problems by representing, adding and subtracting amounts of money.
  - 5.1 solve problems using combinations of coins and bills
  - 5.2 know and use the decimal notation and the dollar and cents symbols for money
6. Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, hundreds and thousands places.
  - 6.1 recognize when an estimate is reasonable in measurements (e.g., closest inch)
7. ***Students identify even and odd numbers to 100***

## ALGEBRA AND FUNCTIONS

1. Students model, represent and interpret number relationships to create and solve problems involving addition and subtraction.
  - 1.1. use the commutative and associative rules to simplify mental calculations and check results
  - 1.2. relate problem situations and number sentences involving addition and subtraction
  - 1.3. solve addition and subtraction problems using data from simple charts, picture graphs and number sentences

## **MEASUREMENT AND GEOMETRY**

1. Students understand that measurement is accomplished by identifying a unit of measure, iterating (repeating) that unit and comparing it to the item to be measured.
  - 1.1. measure the length of objects by iterating (repeating) a non-standard or standard unit
  - 1.2. use different units to measure the same object and predict whether the measure will be greater or smaller when a different unit is used
  - 1.3. measure the length of an object to the nearest inch and/or centimeter
  - 1.4. tell time to the nearest quarter hour *and 5-minute intervals* and know time relationships
2. (e.g., minutes in an hour, days in a month, weeks in year)
  - 2.1. determine the duration of time intervals in hours (e.g., 11:00 a.m. to 4:00 p.m.)
3. Students identify and describe the elements that compose common figures in the plane and common objects in space.
  - 3.1. describe and classify plane and solid geometric shapes (e.g., circle, triangle, square, rectangle, sphere, pyramid, cube, rectangular prism) according to the number and shape of faces, edges and vertices
  - 3.2. put shapes together and take them apart to form other shapes (e.g., two congruent right triangles can form a rectangle)

## **STATISTICS, DATA ANALYSIS AND PROBABILITY**

1. Students collect, record, organize, display and interpret numerical data on bar graphs and other representations.
  - 1.1. record numerical data in systematic ways, keeping track of what/who has been counted
  - 1.2. represent the same data set in more than one way (e.g., charts with tallies, and bar graphs)
  - 1.3. identify features of data sets (range and mode)
  - 1.4. ask and answer simple questions related to data representations
2. Students demonstrate an understanding of patterns and how they grow, and describe them in general ways.
  - 2.1. recognize, describe, extend and explain how to get the next term in linear patterns (e.g., 4, 8, 12 ...; the number of ears on 1 horse, 2 horses, 3 horses, 4 horses)
  - 2.2. solve problems involving simple number patterns

## **MATHEMATICAL REASONING**

1. Students make decisions about how to set up a problem.
  - 1.1. decide about the approach, materials and strategies to use
  - 1.2. use tools such as manipulatives or sketches to model problems
2. Students solve problems and justify their reasoning.
  - 2.1. defend the reasoning used and justify the procedures selected
  - 2.2. make precise calculations and check the validity of the results from the context of the problem
3. Students note connections between one problem and another.

# GRADE 3

By the end of third grade, students deepen their understanding of place value and their understanding of and skill with addition, subtraction, multiplication and division of whole numbers. They estimate, measure and describe objects in space. They use patterns to help solve problems. They represent number relationships and conduct simple probability experiments.

## NUMBER SENSE

1. Students understand place value of whole numbers.
  - 1.1. count, read, and write whole numbers to **100,000**
  - 1.2. compare and order whole numbers to **100,000**
  - 1.3. identify the place value for each digit in numbers to **100,000**
  - 1.4. round off numbers to **100,000** to the nearest ten, hundred, thousand, **and ten thousand**
  - 1.5. use expanded notation to represent numbers (e.g.,  $3,206 = 3,000 + 200 + 6$ )
2. Students calculate and solve problems involving addition, subtraction, multiplication and division.
  - 2.1. find the sum or difference of two whole numbers between 0 and 10,000
  - 2.2. memorize to automaticity the multiplication table for numbers between 1 and **12**
  - 2.3. use the inverse relationship of multiplication and division to compute and check results
  - 2.4. solve simple problems involving multiplication of multi-digit numbers by one-digit numbers ( $3,671 \times 3 = \underline{\quad}$ )
  - 2.5. solve division problems in which a multi-digit number is evenly divided by a one-digit number ( $135/5$ )
  - 2.6. understand the special properties of 0 and 1 in multiplication and division
  - 2.7. determine the unit cost when given the total cost and number of units
  - 2.8. solve problems which combine two or more of the skills above
3. Students understand the relationship between whole numbers, simple fractions and decimals.
  - 3.1. compare fractions represented by drawings or concrete materials to show equivalency, and to add and subtract simple fractions in context (e.g.,  $1/2$  of a pizza is the same amount as  $2/4$  of another pizza that is the same size; show that  $3/8$  is more than  $1/8$ )
  - 3.2. add and subtract simple fractions (e.g., determine that  $1/8 + 3/8$  is the same as  $1/2$ )
  - 3.3. solve problems involving addition, subtraction, multiplication and division of money amounts in decimal notation and multiply and divide money amounts in decimal notation using whole number multipliers and divisors
  - 3.4. know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents is  $1/2$  of a dollar, 75 cents is  $3/4$  of a dollar)

## ALGEBRA AND FUNCTIONS

1. Students select appropriate symbols, operations and properties to represent, describe, simplify and solve simple number relationships.
  - 1.1. represent relationships of quantities in the form of mathematical expressions, equations, or inequalities
  - 1.2. solve problems involving numeric equations or inequalities
  - 1.3. select appropriate operational and relational symbols to make an expression true (e.g.,  $4 \underline{\quad} 3 = 12$ , what operation symbol goes in the blank?)
  - 1.4. express simple unit conversions in symbolic form (e.g., #inches = #feet  $\times$  12)
  - 1.5. recognize and use the commutative and associative properties of multiplication (e.g., if  $5 \times 7 = 35$ , then what is  $7 \times 5$ ?, if  $5 \times 7 \times 3 = 105$ , then what is  $7 \times 3 \times 5$ ?)
2. Students represent simple functional relationships.
  - 2.1. solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the per unit cost)
  - 2.2. extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses can be calculated by counting by 4s or by multiplying the number of horses by 4)



## MEASUREMENT AND GEOMETRY

1. Students choose and use appropriate units and measurement tools to quantify the properties of objects.
  - 1.1. choose appropriate units (metric and U.S. customary) and tools, and estimate and measure length, liquid volume and weight/mass
  - 1.2. estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them
  - 1.3. find the perimeter of a polygon with integer sides
  - 1.4. carry out simple unit conversions within a system of measurement (e.g., centimeters and meters, hours and minutes)
  - 1.5. *tell time to the nearest minute and know time relationships (e.g. half-past, quarter 'til)*
2. Students describe and compare the attributes of plane and solid geometric figures and use their understanding to show relationships and solve problems.
  - 2.1. identify, describe and classify polygons (including pentagons, hexagons and octagons)
  - 2.2. identify attributes of triangles (e.g., two equal sides for the isosceles triangle, three equal sides for the equilateral triangle, right angle for the right triangle)
  - 2.3. identify attributes of quadrilaterals (e.g., parallel sides for the parallelogram, right angles for the rectangle, equal sides and right angles for the square)
  - 2.4. identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle
  - 2.5. identify, describe, and classify common three-dimensional geometric objects (e.g., cube, rectangular solid, sphere, prism, pyramid, cone, cylinder)
  - 2.6. identify the common solid objects that are the component parts needed to make a more complex solid object

## STATISTICS, DATA ANALYSIS and PROBABILITY

1. Students conduct simple probability experiments by determining the number of possible outcomes, and make simple predictions.
  - 1.1. identify whether common events are certain, likely, unlikely, or improbable
  - 1.2. record the possible outcomes for a simple event (e.g., tossing a coin) and systematically keep track of the outcomes when the event is repeated many times
  - 1.3. summarize and display the results of probability experiments in a clear and organized way (e.g., use a bar graph or a line plot)
  - 1.4. use the results of probability experiments to predict future events (e.g., use a line plot to predict the temperature forecast for the next day)

## MATHEMATICAL REASONING

1. Students make decisions about how to approach problems.
  - 1.1. analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing and prioritizing information, and observing patterns
  - 1.2. determine when and how to break a problem into simpler part
2. Students use strategies, skills and concepts in finding solutions.
  - 2.1. use estimation to verify the reasonableness of calculated results
  - 2.2. apply strategies and results from simpler problems to more complex problems
  - 2.3. use a variety of methods such as words, numbers, symbols, charts, graphs, tables, diagrams and models to explain mathematical reasoning
  - 2.4. express the solution clearly and logically using appropriate mathematical notation and terms and clear language, and support solutions with evidence, in both verbal and symbolic work
  - 2.5. indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy
  - 2.6. make precise calculations and check the validity of the results from the context of the problem
3. Students move beyond a particular problem by generalizing to other situations.

- 3.1. evaluate the reasonableness of the solution in the context of the original situation
- 3.2. note method of deriving the solution and demonstrate conceptual understanding of the derivation by solving similar problems
- 3.3. develop generalizations of the results obtained and extend them to other circumstances (*real-life applications*)

# GRADE 4

By the end of fourth grade, students understand large numbers and addition, subtraction, multiplication and division of whole numbers. They describe and compare simple fractions and decimals. They understand the properties of and the relationships between plane geometric figures. They collect, represent and analyze data to answer questions.

## NUMBER SENSE

1. Students understand place value of whole numbers and decimals to two decimal places, how these relate to simple fractions, and use concepts of negative numbers.
  - 1.1. read and write whole numbers in the millions
  - 1.2. order and compare whole numbers and decimals to two decimal places
  - 1.3. round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand or hundred thousand
  - 1.4. decide when *an estimated and/or* rounded solution is called for, and explain why this is the case
  - 1.5. interpret different meanings for fractions including parts of a whole, parts of a set, indicated division of whole numbers and quantities (and measures) between whole numbers on a number line; and relate to simple decimals on a number line
  - 1.6. write tenths and hundredths in decimal and fraction notation and know fraction/decimal equivalents for halves and fourths (e.g.,  $1/2 = 0.5$  or  $.50$ ;  $7/4 = 1\ 3/4 = 1.75$ )
  - 1.7. write the fraction represented by a drawing of parts of a figure; represent a given fraction using drawings
  - 1.8. use concepts of negative numbers (e.g., on a number line, in counting, in temperature, "owing")
  - 1.9. identify the relative position of fractions, mixed numbers, and decimals to two decimal places on the number line
  - 1.10. *tell time to the nearest second and know time relationships*
2. Students extend their use and understanding of whole numbers to addition and subtraction of simple decimals.
  - 2.1. estimate and compute the sum or difference of whole numbers and positive decimals to two places
  - 2.2. round two place decimals to one decimal or the nearest whole number, and use rounding to judge the reasonableness of an answer
3. Students solve problems involving addition, subtraction, multiplication and division of whole numbers, including the addition and subtraction of negative numbers, and understand the relationships among the operations.
  - 3.1. develop quick recall of multiplication facts and related division facts.
  - 3.2. continue to develop fluency with whole number multiplication
  - 3.3. demonstrate understanding of, and the ability to use standard algorithms for addition and subtraction of multi-digit numbers
  - 3.4. demonstrate understanding of, and ability to use standard algorithms for multiplying a multi-digit number by a two digit number and long division for dividing a multi-digit number by a one digit number; use relationships between them to simplify computations and to check results
  - 3.5. solve problems involving multiplication of multi-digit numbers by two-digit numbers
  - 3.6. solve problems involving division of multi-digit numbers by one-digit numbers
4. Students know how to factor small whole numbers.
  - 4.1. understand that many whole numbers decompose in different ways
5. (e.g.,  $12 = 4 \times 3 = 2 \times 6 = 2 \times 2 \times 3$ )
  - 5.1. know that numbers such as 2, 3, 5, 7, 11 do not have any factors except 1 and themselves, and that such numbers are called prime numbers

## ALGEBRA AND FUNCTIONS

1. Students use and interpret variables, mathematical symbols and properties to write and simplify expressions and sentences.
  - 1.1. use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate understanding and use of a concept of a variable)

- 1.2. interpret and evaluate mathematical expressions that use parentheses
  - 1.3. use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations
  - 1.4. use and interpret formulas (e.g., Area = length times width or  $A = lw$ ) to answer questions about quantities and their relationships
  - 1.5. understand that an equation such as  $y = 3x + 5$  is a prescription for determining a second number when a first number is given
2. Students know how to manipulate equations.
    - 2.1. know and understand that equal added to equals are equal
    - 2.2. know and understand that equals multiplied by equals are equal

## MEASUREMENT AND GEOMETRY

1. Students understand perimeter and area.
  - 1.1. measure the area of rectangular shapes, using appropriate units square centimeter<sup>2</sup>, square meter<sup>2</sup>, square kilometer<sup>2</sup>, square inches<sup>2</sup>, square yard<sup>2</sup>, square mile<sup>2</sup>
  - 1.2. recognize that the rectangles having the same area can have different perimeters
  - 1.3. understand that the same number can be the perimeter of different rectangles, each having a different area
  - 1.4. understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use these formulas to find the areas of more complex figures by dividing them into parts with these basic shapes
2. Students use two-dimensional coordinate grids to represent points and graph lines and simple figures.
  - 2.1. **know how to write ordered pairs and** draw the points corresponding to linear relationships on graph paper (e.g., draw the first ten points for the equation  $y = 3x$  and connect them using a straight line)
  - 2.2. understand that the length of a horizontal line segment equals the difference of the x-coordinates
  - 2.3. understand that the length of a vertical line segment equals the difference of the y-coordinates
3. Students demonstrate understanding of plane and solid geometric objects. They use this knowledge to show relationships and solve problems.
  - 3.1. identify lines that are parallel and perpendicular
  - 3.2. identify the radius and diameter of a circle
  - 3.3. identify congruent figures
  - 3.4. identify figures that have bilateral and rotational symmetry
  - 3.5. know the definitions of right angle, acute angle and obtuse angle. They understand that 90, 180, 270, and 360 degrees are, respectively, associated with  $1/4$ ,  $1/2$ ,  $3/4$  and full turns.
  - 3.6. visualize, describe and represent geometric solids (e.g., prisms, pyramids, etc.) in terms of the number and shape of faces, edges and vertices; interpret two-dimensional representations of three-dimensional objects; and draw patterns (of faces) for a solid that when folded will make a model of the solid
  - 3.7. know the definitions of different triangles (e.g., equilateral, isosceles, scalene) and identify their features
  - 3.8. know the definition of different quadrilaterals (e.g., rhombus, square, rectangle, parallelogram, trapezoid)

## STATISTICS, DATA ANALYSIS and PROBABILITY

1. Students organize, represent and interpret numerical and categorical data, and clearly communicate their findings.
  - 1.1. formulate survey questions, systematically collect and represent data on a number line, and coordinate graphs, tables and charts
  - 1.2. identify the mode(s) for sets of categorical data, and the mode(s), median, and any apparent outliers for numerical data sets
  - 1.3. interpret one- and two-variable data graphs to answer questions about a situation
2. Students make predictions for simple probability situations.
  - 2.1. represent all possible outcomes for a simple probability situation in an organized way (e.g., tables, grids, tree diagrams)
  - 2.2. express outcomes of experimental probability situations verbally and numerically (e.g., 3 out of 4;  $3/4$ )

## **MATHEMATICAL REASONING**

1. Students make decisions about how to approach problems.
  - 1.1. analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing and prioritizing information, and observing patterns
  - 1.2. determine when and how to break a problem into simpler parts
2. Students use strategies, skills and concepts in finding solutions.
  - 2.1. use estimation to verify the reasonableness of calculated results
  - 2.2. apply strategies and results from simpler problems to more complex problems
  - 2.3. use a variety of methods such as words, numbers, symbols, charts, graphs, tables, diagrams and models to explain mathematical reasoning
  - 2.4. express the solution clearly and logically using appropriate mathematical notation and terms and clear language, and support solutions with evidence, in both verbal and symbolic work
  - 2.5. indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy
  - 2.6. make precise calculations and check the validity of the results from the context of the problem
3. Students move beyond a particular problem by generalizing to other situations.
  - 3.1. evaluate the reasonableness of the solution in the context of the original situation
  - 3.2. note method of deriving the solution and demonstrate conceptual understanding of the derivation by solving similar problems
  - 3.3. develop generalizations of the results obtained and extend them to other circumstances

# GRADE 5

By the end of fifth grade, students increase their facility with the four basic arithmetic operations applied to positive and negative numbers, fractions and decimals. They know and use common measuring units to determine length and area; they know and use formulas to determine the volume of simple geometric figures. Students know the concept of angle measurement and use a protractor and compass in solving problems. They use grids, tables, graphs, and charts to record and analyze data.

## NUMBER SENSE

1. Students compute with very large and very small numbers, positive and negative numbers, decimals and fractions and understand the relationship between decimals, fractions and percents. They understand the relative magnitudes of numbers.
  - 1.1. estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers
  - 1.2. interpret percents as part of a hundred; find decimal and percent equivalents for common fractions; explain why they represent the same value; and compute a given percent of a whole number
  - 1.3. understand and compute positive integer powers of non-negative integers; compute examples as repeated multiplication
  - 1.4. determine the prime factors of all numbers through 50 and write numbers as the product of their prime factors using exponents to show multiples of a factor (e.g.,  $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$ )
  - 1.5. identify and represent positive and negative integers, decimals, fractions and mixed numbers on a number line
2. Students perform calculations and solve problems involving addition, subtraction and simple multiplication and division of fractions and decimals.
  - 2.1. add, subtract, multiply and divide with decimals and negative numbers and verify the reasonableness of the results
  - 2.2. are proficient with division, including division with positive decimals and long division with multiple digit divisors ***and the adding of "0" to the dividend to reach a terminating decimal***
  - 2.3. solve simple problems including ones arising in concrete situations involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less) and express answers in simplest form
  - 2.4. understand the concept of multiplication and division of fractions
  - 2.5. compute and perform simple multiplication and division of fractions and apply these procedures to solving problems

## ALGEBRA AND FUNCTIONS

1. Students use variables in simple expressions, compute the value of the expression for specific values of the variable, and plot and interpret the results.
  - 1.1. use information taken from a graph or equation to answer questions about a problem situation
  - 1.2. use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution
  - 1.3. know and use the distributive property in equations and expressions with variables
  - 1.4. identify and graph ordered pairs in the four quadrants of the coordinate plane
  - 1.5. solve problems involving linear functions with integer values, write the equation, and graph the resulting ordered pairs of integers on a grid

## MEASUREMENT AND GEOMETRY

1. Students understand and compute volumes and areas of simple objects.
  - 1.1. derive and use the formula for the area of right triangles and of parallelograms by comparing with the area of rectangles (i.e., two of the same triangles make a rectangle with twice the area; a parallelogram is compared to a rectangle with the same area found by cutting and pasting a right triangle)
  - 1.2. construct cube and rectangular boxes from two-dimensional patterns and use this to compute the surface area for these objects

1.3. understand the concept of volume and use the appropriate units in common measuring systems (cubic centimeter<sup>3</sup>, cubic meter<sup>3</sup>, cubic inches<sup>3</sup>, cubic yard<sup>3</sup>) to compute the volume of rectangular solids

1.4 differentiate between and use appropriate units of measures for, two- and three-dimensional objects (perimeter, area and volume)

1.4. *understand and solve problems using elapsed time*

2. Students identify, describe, draw and classify properties of, and relationships between, plane and solid geometric figures.
  - 2.1. measure, identify and draw angles, perpendicular and parallel lines, *lines, rays, line segments*, rectangles and triangles, using appropriate tools (e.g., straight edge, ruler, compass, protractor and drawing software)
  - 2.2. know that the sum of the angles of any triangle is 180 and the sum of the angles of any quadrilateral is 360 and use this information to solve problems
  - 2.3. visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids

## **STATISTICS, DATA ANALYSIS and PROBABILITY**

1. Students display, analyze, compare and interpret different data sets, including data sets that are not the same size.
  - 1.1. know the concepts of mean, median, and mode; compute and compare them in simple examples and notice that they can differ
  - 1.2. organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for different kinds of data sets
  - 1.3. use fractions and percentages to compare data sets of different size
  - 1.4. identify ordered pairs of data from a graph and interpret the meaning of the data in terms of the situation depicted by the graph
  - 1.5. know how to write ordered pairs correctly (e.g., (x, y))

## **MATHEMATICAL REASONING**

1. Students make decisions about how to approach problems.
  - 1.1. analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing and prioritizing and observing patterns
  - 1.2. determine when and how to break a problem into simpler parts
2. Students use strategies, skills and concepts in finding solutions.
  - 2.1. use estimation to verify the reasonableness of calculated results
  - 2.2. apply strategies and results from simpler problems to more complex problems
  - 2.3. use a variety of methods such as words, numbers, symbols, charts, graphs, tables, diagrams and models to explain mathematical reasoning
  - 2.4. express the solution clearly and logically using appropriate mathematical notation and terms and clear language, and support solutions with evidence, in both verbal and symbolic work
  - 2.5. indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy
  - 2.6. make precise calculations and check the validity of the results from the context of the problem
3. Students move beyond a particular problem by generalizing to other situations.
  - 3.1. evaluate the reasonableness of the solution in the context of the original situation
  - 3.2. note method of deriving the solution and demonstrate conceptual understanding of the derivation by solving similar problems
  - 3.3. develop generalizations of the results obtained and extend them to other circumstances

# GRADE 6

By the end of sixth grade, students have mastered the four arithmetic operations with positive and negative numbers, whole numbers, fractions and decimals; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concept of and how to calculate the range, mean, median and mode of data sets. They analyze data and sampling processes for possible bias and misleading conclusions, and they use addition and multiplication of fractions routinely to calculate probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about  $\pi$  and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in representing an unknown part of a ratio. They solve 1-step linear equations.

## NUMBER SENSE

1. Students compare and order fractions, decimals, and mixed numbers. They solve problems involving fractions, ratios, proportions, and percentages.
  - 1.1. compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line
  - 1.2. interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities using appropriate notations ( $a/b$ ,  $a$  to  $b$ ,  $a:b$ )
  - 1.3. use proportions to solve problems (e.g., determine the value of  $N$  if  $4/7 = N/21$ , find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as multiplication of both sides of an equation by a multiplicative inverse.
  - 1.4. calculate given percentages of quantities and solve problems involving discounts at sales, interest earned and tips
2. Students calculate and solve problems involving addition, subtraction, multiplication and division of rational numbers.
  - 2.1. solve problems involving addition, subtraction, multiplication and division of fractions and explain why a particular operation was used for a given situation
  - 2.2. explain the meaning of multiplication and division of fractions and perform the calculations (e.g.,  $5/8$  divided by  $15/16 = 5/8 \times 16/15 = 2/3$ )
  - 2.3. solve addition, subtraction, multiplication and division problems, including those arising in concrete situations that use positive and negative numbers and combinations of these operations
  - 2.4. determine the least common multiple and greatest common divisor of whole numbers. Use them to solve problems with fractions (e.g., to find a common denominator in order to add two fractions or to find the reduced form for a fraction)

## ALGEBRA AND FUNCTIONS

1. Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations and graph and interpret their results.
  - 1.1. write and solve one-step linear equations in one variable
  - 1.2. write and evaluate an algebraic expression for a given situation using up to three variables
  - 1.3. apply algebraic order of operations and the commutative, associative and distributive properties to evaluate expressions and justify each step in the process
  - 1.4. solve problems using correct order of operations manually and by using a scientific calculator
2. Students analyze and use tables, graphs and rules to solve problems involving rates and proportions.
  - 2.1. convert from one unit of measurement to another (e.g., from feet to miles, from centimeters to inches, *from meters to kilometers*)
  - 2.2. demonstrate understanding that rate is a measure of one quantity per unit value of another quantity
  - 2.3. solve problems involving rates, average speed, distance and time
3. Students investigate geometric patterns and describe them algebraically.
  - 3.1. use variables in expressions describing geometric quantities (e.g.,  $P = 2w + 2l$ ,  $A = 1/2 bh$ ,  $C = \pi d$ , which give the perimeter of a rectangle, area of a triangle, and circumference of a circle, respectively)



- 3.2. express simple relationships arising from geometry in symbolic form

## **MEASUREMENT AND GEOMETRY**

1. Students deepen their understanding of measurement of plane and solid shapes and use this understanding to solve problems.
  - 1.1. understand the concept of a constant number like  $\pi$ . Know *and memorize* the formula for the circumference and area of a circle
  - 1.2. know common estimates of  $\pi$  (3.14; 22/7) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements
  - 1.3. know and use the formulas for the volume of triangular prisms and cylinders (area of base  $\times$  height); compare and explain the similarity between these formulas and the formula for the volume of a rectangular solid
2. Students identify and describe the properties of two-dimensional figures.
  - 2.1. identify angles as vertical, adjacent, complementary and/or supplementary and provide descriptions of these terms
  - 2.2. use the properties of complimentary and supplementary angles and of the angles of a triangle to solve problems involving an unknown angle
  - 2.3. draw quadrilaterals and triangles given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle)

## **STATISTICS, DATA ANALYSIS and PROBABILITY**

1. Students compute and analyze statistical measurement for data sets.
  - 1.1. compute the range, mean, median and mode of data sets
  - 1.2. understand how additional data added to data sets can effect these computations of measures of central tendency
  - 1.3. understand how the inclusion or exclusion of outliers affect measures of central tendency
  - 1.4. know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context
2. Students use data samples of a population and describe the characteristics and limitations of the samples.
  - 2.1. compare different samples from a population with the data from the entire population and identify when it makes sense to use a sample
  - 2.2. identify different ways of selecting a sample (e.g., convenience sampling, those who respond to a survey, random sampling) and which makes a sample more representative for a population
  - 2.3. analyze data displays and explain how the way the question was asked might have influenced the results obtained, and/or how the way the results were displayed might have influenced the conclusions reached
  - 2.4. identify data that represent sampling and explain why the sample (and the display) may be biased
  - 2.5. identify claims based on statistical data and, in simple cases, evaluate the validity of the claims
3. Students determine theoretical and experimental probabilities and use these to make predictions about events.
  - 3.1. represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome
  - 3.2. use data to estimate the probability for future events (e.g., batting averages or number of accidents per mile driven)
  - 3.3. represent probabilities as ratios, proportions, and decimals between 0 and 1, and percents between 0 and 100 and check that probabilities computed are reasonable; know how this is related to the probability of an event not occurring
  - 3.4. understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities
  - 3.5. understand the difference between independent and dependent events and how this affects the results for specific probability situations

## **MATHEMATICAL REASONING**

1. Students make decisions about how to approach problems.
  - 1.1. analyze problems by identifying relationships, discriminating relevant from irrelevant information, identifying missing information, sequencing and prioritizing information and observing patterns
  - 1.2. formulate and justify mathematical conjectures based upon a general description of the mathematical question or problem posed
  - 1.3. determine when and how to break a problem into simpler parts
2. Students use strategies, skills and concepts in finding solutions
  - 2.1. use estimation to verify the reasonableness of calculated results
  - 2.2. apply strategies and results from simpler problems to more complex problems
  - 2.3. estimate unknown quantities graphically and solve for them using logical reasoning, and arithmetic and algebraic techniques
  - 2.4. use a variety of methods such as words, numbers, symbols, charts, graphs, tables, diagrams and models to explain mathematical reasoning
  - 2.5. express the solution clearly and logically using appropriate mathematical notation and terms and clear language, and support solutions with evidence, in both verbal and symbolic work
  - 2.6. indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy
  - 2.7. make precise calculations and check the validity of the results from the context of the problem
3. Students move beyond a particular problem by generalizing to other situations.
  - 3.1. evaluate the reasonableness of the solution in the context of the original situation
  - 3.2. note method of deriving the solution and demonstrate conceptual understanding of the derivation by solving similar problems
  - 3.3. develop generalizations of the results obtained and the strategies used and extend them to new problem situations

# GRADE 7

By the end of seventh grade students are adept at manipulating numbers and equations and understand the general principles at work. They understand and use factoring of numerator and denominators and properties of exponents. They know the Pythagorean Theorem and solve problems where they compute the length of an unknown side. Students know how to compute the surface area and volume of basic 3-D objects and understand how they change under a change in scale. Students convert between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percent) and are proficient at changing from one to another. They increase their facility with ratio and proportion and compute percentages of increase and decrease and simple compound interest. They graph linear functions and understand the idea of slope and its relation to ratio.

## NUMBER SENSE

1. Students know the properties of and compute with rational numbers expressed in a variety of forms.
  - 1.1. read, write and compare rational numbers in scientific notation (positive and negative powers of 10), approximate numbers using scientific notation
  - 1.2. add, subtract, multiply and divide rational numbers, integers, fractions and decimals and take rational numbers to whole number powers
  - 1.3. convert fractions to decimals and percents and use these representations in estimation, computation and applications
  - 1.4. differentiate between rational and irrational numbers
  - 1.5. know that every fraction is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions
  - 1.6. calculate percent of increases and decreases of a quantity
  - 1.7. solve problems that involve discounts, markups, commissions, profit and simple compound interest
2. Students use exponents, powers, and roots and use exponents in working with fractions.
  - 2.1. understand negative whole number exponents. Multiply and divide expressions involving exponents with a common base
  - 2.2. add and subtract fractions using factoring to find common denominators
  - 2.3. multiply, divide, and simplify fractions using exponent rules
  - 2.4. use the inverse relationship between raising to a power and root extraction for perfect square integers; and, for integers which are not square, determine without a calculator, the two integers between which its square root lies, and explain why
  - 2.5. understand the meaning of the absolute value of a number, interpret it as the distance of the number from zero on a number line and determine the absolute value of real numbers

## ALGEBRA AND FUNCTIONS

1. Students express quantitative relationships using algebraic terminology, expressions, equations, inequalities and their graphs.
  - 1.1. use variables and appropriate operations to write an expression, equation, inequality, or system of equations or inequalities which represent a verbal description (e.g., three less than a number, half as large as area A)
  - 1.2. use order of operations correctly to evaluate algebraic expressions such as  $3(2x + 5)^2$
  - 1.3. simplify numerical expressions by applying properties of rational numbers (identity, inverse, distributive, associative, commutative), and justify the process used
  - 1.4. use algebraic terminology correctly (e.g., variable, equation, term, coefficient, inequality, expression, constant)
  - 1.5. represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in terms of the situation represented by the graph
2. Students interpret and evaluate expressions involving integer powers and simple roots.
  - 2.1. interpret positive whole number powers as repeated multiplication and negative whole numbers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

- 2.2. 2.2 multiply and divide monomials; extend the process of taking powers and extracting roots to monomials, when the latter results in a monomial with an integer exponent
3. Students graph and interpret linear and some non-linear functions.
  - 3.1. graph functions of the form  $y = nx^2$  and  $y = nx^3$  and use in solving problems
  - 3.2. plot the values from the volumes of a 3-D shape for various values of its edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and a varying length equilateral triangle base)
  - 3.3. graph linear functions, noting that the vertical change (change in y-value) per unit horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph
  - 3.4. plot values of the quantities whose ratio is always the same (cost vs. number of an item, feet vs. inches, circumference vs. diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.
4. Students solve simple linear equations and inequalities over the rational numbers.
  - 4.1. solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution(s) in terms of the context from which they arose and verify the reasonableness of the results
  - 4.2. solve multi-step problems involving rate, average speed, distance and time, or direct variation

## MEASUREMENT AND GEOMETRY

1. Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems.
  - 1.1. compare weights, capacities, geometric measures, times and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters)
  - 1.2. construct and read scale drawings and models
  - 1.3. use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems, checking units of the solutions; and use dimensional analysis to check the reasonableness of the answer
2. Students compute the perimeter, area and volume of common geometric objects and use these to find measures of less common objects; they know how perimeter, area, and volume are affected under changes of scale.
  - 2.1. routinely use formulas for finding the perimeter and areas of basic two-dimensional figures and for the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and circular cylinders
  - 2.2. estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking them up into more basic geometric objects
  - 2.3. compute the length of the perimeter, the surface area of the faces, and the volume of a 3-D object built from rectangular solids. They understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor
  - 2.4. relate the changes in measurement under change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 12 square inches, 1 cubic inch = 2.54 cubic centimeters)
3. Students know the Pythagorean Theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures.
  - 3.1. identify and construct basic elements of geometric figures, (e.g., altitudes, midpoints, diagonals, angle bisectors and perpendicular bisectors; and central angles, radii, diameters and chords of circles) using compass and straight-edge
  - 3.2. understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections
  - 3.3. know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right triangle and lengths of other line segments, and, in some situations, empirically verify the Pythagorean Theorem by direct measurement
  - 3.4. demonstrate an understanding of when two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures
  - 3.5. construct two-dimensional patterns for three-dimensional models such as cylinders, prisms and cones

- 3.6. identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and how two or more objects are related in space (e.g., skew lines, the possible ways three planes could intersect)

## **STATISTICS, DATA ANALYSIS AND PROBABILITY**

1. Students collect, organize and represent data sets that have one or more variables and identify relationships among variables within a data set both manually and by using an electronic spreadsheet program.
  - 1.1. know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use them to display a single set of data or compare two sets of data
  - 1.2. represent two numerical variables on a scatter plot and informally describe how the data points are distributed and whether there is an apparent relationship between the two variables (e.g., time spent on homework and grade level)
  - 1.3. understand the meaning of and be able to compute the minimum, the lower quartile, the median, the upper quartile and the maximum of a data set

## **MATHEMATICAL REASONING**

1. Students make decisions about how to approach problems.
  - 1.1. analyze problems by identifying relationships, discriminating relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns
  - 1.2. formulate and justify mathematical conjectures based upon a general description of the mathematical question or problem posed
  - 1.3. determine when and how to break a problem into simpler parts
2. 2.1 use estimation to verify the reasonableness of calculated results
3. 2.2 apply strategies and results from simpler problems to more complex problems
  - 3.1. estimate unknown quantities graphically and solve for them using logical reasoning, and arithmetic and algebraic techniques
  - 3.2. make and test conjectures using both inductive and deductive reasoning
  - 3.3. use a variety of methods such as words, numbers, symbols, charts, graphs, tables, diagrams and models to explain mathematical reasoning
  - 3.4. express the solution clearly and logically using appropriate mathematical notation and terms and clear language, and support solutions with evidence, in both verbal and symbolic work
  - 3.5. indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy
  - 3.6. make precise calculations and check the validity of the results from the context of the problem
4. Students determine a solution is complete and move beyond a particular problem by generalizing to other situations.
5. 3.1 evaluate the reasonableness of the solution in the context of the original situation
6. 3.2 note method of deriving the solution and demonstrate conceptual understanding of the derivation by solving similar problems
7. 3.3 develop generalizations of the results obtained and the strategies used and extend them to new problem situations

# Grade 8-ALGEBRA I

Symbolic reasoning and calculations with symbols are central in algebra. In the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem solving situations.

1. Students identify and use the arithmetic properties of subsets of integers, rational, irrational and real numbers. This includes closure properties for the four basic arithmetic operations where applicable.
  - 1.1. Students use properties of numbers to demonstrate that assertions are true or false.
2. Students understand and use such operations as taking the opposite, reciprocal, raising to a power, and taking a root. This includes the understanding and use of the rules of exponents.
3. Students solve equations and inequalities involving absolute values.
4. Students simplify expressions prior to solving linear equations and inequalities in one variable such as  $3(2x-5) + 4(x-2) = 12$ .
5. Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable, with justification of each step.
6. Students graph a linear equation, and compute the x- and y- intercepts (e.g., graph  $2x + 6y = 4$ ). They are also able to sketch the region defined by linear inequality (e.g., sketch the region defined by  $2x + 6y < 4$ ).
7. Students verify that a point lies on a line given an equation of the line. Students are able to derive linear equations using the point-slope formula.
8. Students understand the concepts of parallel and perpendicular lines and how their slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.
9. Students solve a system of two linear equations in two variables algebraically, and are able to interpret the answer graphically. Students are able to use this to solve a system of two linear inequalities in two variables, and to sketch the solution sets.
10. Students add, subtract, multiply and divide monomials and polynomials. Students solve multistep problems, including word problems, using these techniques.
11. Students apply basic factoring techniques to second and simple third degree polynomials. These techniques include finding a common factor to all of the terms in a polynomial and recognizing the difference of two squares, and recognizing perfect squares of binomials.
12. Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing to lowest terms.
13. Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems using these techniques.
14. Students solve a quadratic equation by factoring or completing the square.
15. Students apply algebraic techniques to rate problems, work problems, and percent mixture problems.
16. Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17. Students determine the domain of independent variables, and range of dependent variables defined by a graph, a set of ordered pairs, or symbolic expression.
18. Students determine whether a relation defined by a graph, a set of ordered pairs, or symbolic expression is a function and justify the conclusion.
19. Students know the quadratic formula and are familiar with its proof by completing the square.
20. Students use the quadratic formula to find the roots of a second degree polynomial and to solve quadratic equations.
21. Students graph quadratic functions and know that their roots are the x-intercepts.
22. Students use the quadratic formula and/or factoring techniques to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.
23. Students apply quadratic equations to physical problems such as the motion of an object under the force of gravity.
24. Students use and know simple aspects of a logical argument.
  - 24.1. Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.
  - 24.2. Students identify the hypothesis and conclusion in logical deduction.
  - 24.3. Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

25. Students use properties of the number system to judge the validity of results, to justify each step of a procedure and to prove or disprove statements.
  - 25.1. Students use properties of numbers to construct simple valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.
  - 25.2. Students judge the validity of an argument based on whether the properties of the real number system and order of operations have been applied correctly at each step.
  - 25.3. Given a specific algebraic statement involving linear, quadratic or absolute value expressions, equations or inequalities, students determine if the statement is true sometimes, always, or never.
26. ***Once able to perfect a skill numerically, students will be able to explain it verbally.***
27. ***The students will translate from verbal to symbolic work.***